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Restored Symmetries, Quark Puzzle, and the Pomeron as a Josephson Current

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Abstract

A special type of symmetry is studied, wherein manifest invariance is restored by direct integration over a set of spontaneously broken ground states. In addition to invariant states and multiplets these symmetry realizations are shown to lead, in general, to clustering effects and quantum supercurrents. A systematic exploration of these symmetry realizations is proposed, mostly in physical situations where it has so far been believed that the only consequences of the symmetry are invariant states and multiplets. An application of these ideas to the quark system yields a possible explanation for the unobservability of free quarks and an interpretation of the Pomeron as a generalized Josephson current. Furthermore, the "narrowing gap mechanism" suggests an explanation for the behavior of the $e^+ e^- \to$ hadrons cross section and a speculation on an approaching phase transition in hadronic production and the observation of free quarks.

1. *Introduction*

Although this point has already been raised by other authors (see, for example, Coleman, 1971) it is always appropriate to emphasize the misleading nature of the name "spontaneously broken symmetry" (SBS). It is misleading because it seems to imply that somehow the symmetry is destroyed or that its consequences are no longer exact, as if we had, for example, added symmetry-breaking terms to a previously symmetric Lagrangian.

A better way to deal with the SBS question is to state that a symmetry of a physical law has at least two distinct ways to realize itself in particular physical systems: (a) the "manifest symmetry" (M-symmetry) way, revealed by the existence of invariant states, degenerate multiplets, and selection rules; (b) the "hidden symmetry" (Coleman, 1971) (H-symmetry) way, revealed by low-energy theorems and Goldstone bosons or Higgs gauge fields. The consequences of the symmetry are quite distinct in the two cases. However, if one deals with an exact symmetry of a physical law, the consequences will be equally exact in both cases.

One of the purposes of this paper is to propose a systematic study of yet another distinct class of exact symmetry realizations, the (c) "restored symmetries" (R-symmetries). These are symmetry realizations that, we could say, manifest themselves in the space of states in a way that is even more subtle than the H-symmetries. This is because at first sight they may look like M-symmetries, and unless one already knows the equations of motion (in which case we might well do without symmetries at all) one may be misled into disregarding the deeper implications of this class of symmetry realizations.

The R-symmetries are associated with cases where the equations of motion are such that one might in principle have the "spontaneous symmetry-breaking" mechanism, but where somehow the symmetry is "restored" after being "spontaneously broken." In the mathematical models that represent this situation the restoration mechanism is described either by (i) choosing one among the set of degenerate ground states and imposing a restriction on the algebra of observables to mask the asymmetry of the ground state, or by (ii) constructing, out of the set of degenerate ground states, an invariant state by an appropriately weighed direct sum (integral). In any case one ends up with a situation that, on the outside level of physical observation, looks so manifestly symmetric that one may be misled into thinking that a M-symmetry is at work and neglect the deeper implications of the R-symmetries.

Examples of R-symmetries have already appeared in the literature. The classical example is Haag's solution (Haag, 1962) of the BCS model in the "sharp particle number" space. Similar examples are contained in papers by Frishman (1967) and Lopuszanski and Reeh (1965, 1966). For some reason, however, to this author's knowledge, these symmetries were never identified as a class distinct from the H-symmetries. This is probably a consequence of their common association to the SBS mechanism. In fact, they are quite distinct, for whereas the natural domain of application of H-symmetries is in the search for an exact symmetrical interpretation of nonsymmetrical situations in nature, the domain of application of R-symmetries is to situations where nature looks obviously symmetric but also contains features that, having, apparently, nothing to do with the symmetries, might in fact be their consequences, if the symmetries are restored instead of manifest.

Once this is realized we might ask ourselves whether there are particular physical situations where such sacred symmetries as rotational and Lorentz invariance, which look so manifest, might not be "spontaneously broken and restored," instead of "manifest." This would be a very interesting situation because of the particular features of the R-symmetries, which in addition to invariant states and multiplets, which are features common to manifest symmetries, also display, as discussed below, clustering effects in low-lying excitations and exchange of nontrivial vacuum quantum numbers (i.e., generalized Josephson currents).

Let a theory be invariant under a group G and let the states $|\alpha\rangle$ of lowest energy (vacua) be such that

$$g\,|\,\alpha\rangle \neq |\,\alpha\rangle$$

for some $g \in G$. This spontaneously broken theory may nevertheless be made to look manifestly symmetric by either (i) restricting the algebra of observables to some classes of G tensors (this is the case in the BCS theory of superconductivity, where the BCS ground state is not gauge invariant, but the observables are restricted to gauge-invariant combinations of equal numbers of creation and annihilation operators); or (ii) defining the physical lowest energy state (physical vacuum) to be the following direct integral:

$$|\,\aleph\,\rangle = \frac{1}{V^{1/2}} \int_{\oplus} g\,|\,\alpha\rangle\, d\mu(g) \qquad (1.1)$$

where $\mu(g)$ is the invariant group measure and V, the volume of the group manifold, is included for normalization purposes. The invariance of the group measure implies then

$$g'\,|\,\aleph\,\rangle = |\,\aleph\,\rangle$$

for all $g' \in G$. For the familiar case of the BCS theory the infinite set of degenerate ground states is

$$\{|\,\alpha\rangle = \prod_k \,(V_k + U_k e^{-i\alpha} a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow})\,|\,0\rangle, \qquad \alpha \in [0, 2\pi]\,\}$$

and the $\aleph$ state is

$$|\,\aleph\,\rangle = \frac{1}{\sqrt{2\pi}} \int_{\oplus} \prod_k (V_k + U_k\, e^{-i2\beta}\, a^\dagger_{k\uparrow}\, a^\dagger_{-k\downarrow})\,|\,0\rangle\, d\beta \qquad (1.2)$$

When the $\aleph$ state is used as the ground state, the physical system is endowed with all the properties of manifest symmetries. In addition two other important effects may also be observed: clustering effects and generalized Josephson currents.

2. *Clustering Effects*

These come about when the representation of G spanned by the set of degenerate ground states $|\,\alpha\rangle$ is obtained by the tensor product of simpler representations. Then, the low-lying excitations tend also to transform like tensor products and not like the simpler representations.

It is true that in the familiar example of a superconductor the Green's function has single-particle poles, i.e., $a^\dagger_{k\uparrow}\,|\,\aleph\,\rangle$ is a possible wave function for an excitation. This simply means that one can inject a single particle into a superconductor. A completely different issue is the question of the possible excitations that are obtained by adding energy to a particular ground state. Here the only way to obtain excited states is to break Cooper pairs, and the number of excitations is always even. In particular this implies that $a^\dagger_k\,|\,\aleph\,\rangle$

and $a_k^\dagger a_{-k}^\dagger | \aleph \rangle$, for example, do not belong to systems with the same ground state.

An exact characterization of the possible physical excitations may be obtained by a simple formalism which, exemplified here for the gauge group in the BCS model, is easily generalizable to the case of other symmetry groups. Construct a set $\{|N\rangle\}$ of "reference states" as follows:

$$|N\rangle = \frac{1}{\sqrt{2\pi}} \int_{\oplus} \prod_k (V_k + U_k\, a_{k\uparrow}^\dagger\, a_{-k\downarrow}^\dagger\, e^{-i2\alpha})\,|0\rangle\, e^{iN\alpha}\, d\alpha \qquad (N \neq 0) \quad (2.1)$$

These states share with the $\aleph$-state the property of being rotationally and translationally invariant. They differ, though, on the quantum number associated with the invariant operator of the gauge group, i.e., "the particle number":

$$g(\beta)\,|N\rangle = e^{-iN\beta}\,|N\rangle$$

The physical excitations are then defined to be those whose projections in the reference states are not all identically zero. In fact

$$\langle N | a_{p_1}^\dagger \cdots a_{p_n}^\dagger | \aleph \rangle = 0 \qquad \text{for } n \text{ odd}$$

whereas the projections are not identically zero for n even. For example

$$\langle N | a_{p's'}^\dagger\, a_{ps}^\dagger | \aleph \rangle = -\, U_p V_p \delta_{N,2} \delta_{p,-p'} \delta_{s,-s'}$$

That the excitations are produced in pairs can be checked experimentally by photoexcitation of a superconductor. Nevertheless, once they are produced the excited electrons can in principle be detected separately by measuring, for example, the attenuation of ultrasounds by preexisting excitations. Of course, this observation requires the detection system to be sensitive to the momenta of individual excitations. If, for example, the detection system is sensitive only to momenta much less than the Fermi momentum, only even numbers of excitations will be detected.

In conclusion: clustering effects in low-lying excitations together with kinematical restrictions may cause individual excitations to be essentially unobservable. An example of this mechanism will be discussed in connection with a vacuum state proposed to explain the quark puzzle.

3. *Generalized Josephson Currents*

Given two weakly interacting superconductors of identical material the eigenstates of the Hamiltonian of the composite system are linear combinations $|\Psi\rangle = \Sigma_N\, a_N |\Psi_N\rangle$, $|\Psi_N\rangle$ being elements of a set of "fixed total particle number" state vectors.

$$\{|\Psi_N\rangle\} = \{|2N\rangle_1 | -2N\rangle_2\}, \qquad N = 1, 2, \ldots$$

As a result the number of particles in each superconductor is not a good quantum number, and there will be a quantum flux of Cooper pairs between the superconductors.

In the general case the states analogous to the reference states constructed in equation (2.1) are

$$|cj\rangle = \frac{1}{V^{1/2}} \int_{\oplus} g\,|\alpha\rangle\, D^{(c)}_{\bar{j}j}\,(g^{-1})\, d\mu(g) \tag{3.1}$$

where $D^{(c)}_{\bar{j}j}$ is a matrix representation of the group G. The transformation properties of these states are

$$g\,|\,cj\rangle = |\,cj'\rangle\, D^{(c)}_{j'j}(g)$$

These states differ from the $\aleph$ state in the quantum numbers of the group G but share with it the remaining vacuum quantum numbers.

Given now two systems in interaction, the ground state of the composite system will be a linear combination of vectors in the set

$$\{P_s(\,|\,cj\rangle_1 \otimes |\,c'j'\rangle_2)\}$$

where by $P_s(\,|\rangle_1 \otimes |\rangle_2)$ we mean the G-scalar projection of the tensor product. Not all the vectors in this set will necessarily participate in the linear combination that defines the ground state of the composite system; that will depend on the exact nature of the interaction. In any case, whenever the ground state is a nontrivial linear combination, there will be, between the interacting systems, a quantum flux possessing the nontrivial quantum numbers of the G-group. In the superconductors case G is the gauge group, hence the Josephson current is a charge flux. In an hypothetical case of "restored Lorentz invariance" the generalized Josephson current would carry angular momentum quantum numbers (spin flux).

As I have said before, the main purpose of this paper is to call attention to the R-symmetry idea as a concept quite distinct from the hidden symmetry, mainly in the scope of their applications, and hopefully to trigger a systematic exploration of this type of symmetry realization in cases where previously manifest symmetries seemed to be the answer.

The main power of the study of the symmetry properties of physical systems is the possibility of attaining far-reaching conclusions about their general behaviour without detailed calculations in particular models. In keeping with this philosophy I will restrict myself in the following examples to a discussion of the general insights that can be obtained by combining empirical facts and general symmetry considerations. The results of detailed calculations in particular models in the R-symmetry framework will be reported elsewhere.

4. *Quark Puzzle*

Here I will discuss the possible relevance of the restored symmetry ideas to the quark puzzle, i.e., to the fact that although the classification of hadronic states and the scaling results in deep inelastic electron scattering seem to suggest a model of weakly bound quarks, no such quarks have so far been found.

No attempt will be made to construct an interacting quark-gluon model and hope to find in some appropriate solutions of the model an explanation for the puzzling features of hadronic phenomena. Instead, the empirical facts will be used as an inspiration for the direct construction of an approximate vacuum for the hadronic world. Some consequences of this vacuum state will then be explored.

By analogy with the superconductor case the experimental information that mesons behave like quark–antiquark pairs and baryons like quark triplets leads us to postulate the existence of an infinitely degenerate set of hadronic vacua characterized by

$$\langle\alpha|\bar{q}(x)q(0)|\alpha\rangle \neq 0 \qquad \text{and} \qquad \langle\alpha|q(x)q(y)q(0)|\alpha\rangle \neq 0 \tag{4.1}$$

The physical hadronic vacuum, of course, will be obtained by the restoration procedure applied to the set $\{|\alpha\rangle\}$. The conditions (4.1) force on the $|\alpha\rangle$ state a structure

$$|\alpha\rangle = \prod (V + Uq^\dagger\bar{q}^\dagger + Wq^\dagger q^\dagger q^\dagger + W\bar{q}^\dagger\bar{q}^\dagger\bar{q}^\dagger)|0\rangle$$

The tensor product is taken over the quantum numbers of the quark operators, in particular over their quadrimomenta.

Let us, for the moment, concentrate on the construction of a ground state for the mesonic system, i.e., let us disregard the terms $Wqqq$ and $W\bar{q}\bar{q}\bar{q}$. For $|\alpha\rangle$ to be a quantum field vacuum it should be invariant for space-time translations. Hence, in each pair, the quadrimomenta labels (k) of the q and $\bar{q}$ operators should have opposite signs. For these operators the notation $\mathbf{q}^\dagger(k+)\cdot\mathbf{q}^\dagger(k-)$ will be used. The plus and minus signs refer to positive and negative helicities and the operators have three $SU(3)$ components. Dealing in first approximation with an $SU(3)$-invariant vacuum, the following two possibilities are available:

$$|\alpha\rangle = \prod_k \left[V + \frac{U}{\sqrt{3}}\,\mathbf{q}^\dagger(k+)\cdot\bar{\mathbf{q}}^\dagger(-k+)\right]\left[V + \frac{U}{\sqrt{3}}\,\mathbf{q}^\dagger(k-)\cdot\bar{\mathbf{q}}^\dagger(-k-)\right]|0\rangle \tag{4.2a}$$

$$|\alpha\rangle = \prod_k \left[V + \frac{U}{\sqrt{3}}\,\mathbf{q}^\dagger(k+)\cdot\bar{\mathbf{q}}^\dagger(-k-)\right]\left[V + \frac{U}{\sqrt{3}}\,\mathbf{q}^\dagger(k-)\cdot\bar{\mathbf{q}}^\dagger(-k+)\right]|0\rangle \tag{4.2b}$$

with $V^2 + U^2 = 1$ for normalization.

In (4.2a) the quark–antiquark operators are coupled to zero spin, whereas in (4.2b) the spin projection along the direction **k**, for each pair, equal unity. Hence, in the first case Lorentz invariance is realized as a "manifest symmetry," whereas in the second this symmetry can only be realized as a

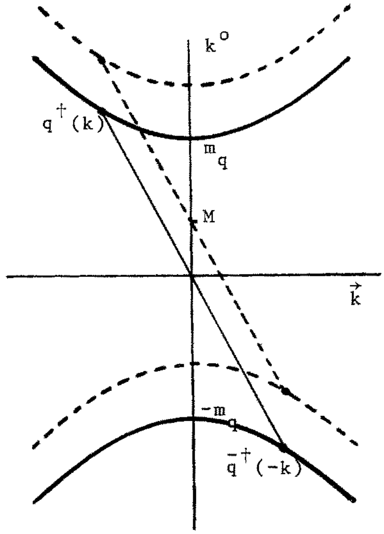

Figure 1 – Virtual quark-antiquark pairs and excitation mechanism.

"restored" one by applying the Lorentz group to $|\alpha\rangle$ and forming the corresponding $\aleph$ state.

The terms $q^{\dagger}(k+)\cdot\bar{q}^{\dagger}(-k-)$ represent virtual quark-antiquark pairs whose total energy and total momentum equal zero, as shown schematically in Figure 1. Obviously, if $|\alpha\rangle$ is to be a vacuum state the interaction must be such that the existence of these pairs lowers the zero-point energy. Hence if one of the pairings is broken the overall energy of the system is raised and a mass M state will be observed (Figure 1). If M is smaller than the effective mass of the quark fields (m_q), the state will consist of a positive-energy quark excitation and a negative-energy antiquark excitation. Physically only those excitations whose energy relative to the vacuum is positive are observable. As a consequence it is only the overall quark-antiquark effect that will be observed, not the individual quark excitations. This is a striking example of how the clustering effects associated with this type of symmetry realizations, plus a kinematical constraint resulting from the translational invariance of the quantum field vacuum, inhibit the observation of elementary excitations.

When $M > m_q$ (see Figure 2) two separate excitations would be observable in case A, but not in case B and if there is a uniform distribution of pairs over the mass shells the probability of case A becomes vanishingly small.

An hadronic vacuum of the form (4.2a) or (4.2b) may thus explain the absence of free quarks in the physical hadronic spectrum.

The choice (4.2b) would imply that in the hadronic system Lorentz symmetry rather than being manifest was of the restored type. Turning to the

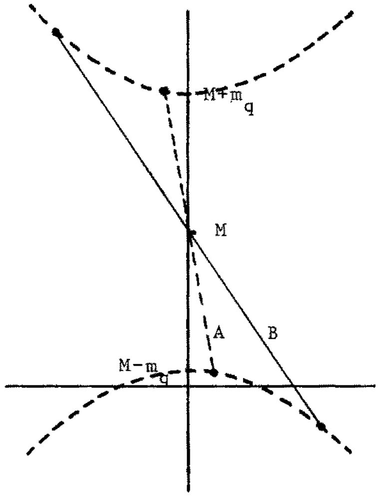

Figure 2—Excitation mechanism when $M > m_q$.

other consequence of this type of symmetry realization, i.e., to the generalized Josephson currents, the conclusion would be that given two hadronic systems in interaction one should expect to have between them a quantum flux with vacuum quantum numbers and spin 1. In elastic scattering this should imply an exchange in the *t*-channel with these quantum numbers but not corresponding to any observed particle. Such an exchange is indeed present and is called the Pomeron. In this light, the Pomeron might thus be interpreted as a generalized Josephson current associated with restored Lorentz invariance.

5. *Baryons*

From the analysis of the meson case we have learned that an approximate description of the vacuum state may be obtained, in the $SU3$ limit, by constructing an α state with $SU3$ realized as a manifest symmetry and Lorentz invariance realized as a restored symmetry. Using this information we may now proceed to the construction of quark operator clusters with nonzero baryon number.

The tensor product qq leads to $\overline{3} \oplus 6$; hence if one insists in having $SU3$ realized as a manifest symmetry this cluster is not acceptable. In qqq, however, one has an $SU3$ scalar, $\epsilon_{ijk}\, q_i q_j q_k$, which is then a possible ground-state cluster. The sum of the quadrimomenta associated to these virtual ground-state clusters should vanish. By the same arguments as before low-lying excitations will behave like three quarks without individual elementary excitations being observed.

Of course, for the three-quark clusters to be present in the vacuum state it is necessary to have a quark–quark attractive interaction, to make the cluster energetically favorable. The important point to notice, though, is the fact that this attractive interaction does not imply the existence of two-quark clusters if $SU3$ is to be realized as a manifest symmetry.

Concerning the question of Lorentz invariance one observes that, in the nonrelativistic limit, the quark operator spins couple to 1/2 or 3/2. Hence, Lorentz invariance can only be realized as a restored symmetry. An immediate consequence as far as generalized Josephson currents are concerned is that one should also expect half-integer spin vacuum exchanges corresponding to the exchange of virtual $(qqq+\bar{q}\bar{q}\bar{q})$ clusters—i.e., in addition to the Pomeron, another diffractive component may be expected, corresponding to a vacuum trajectory of half-integer intersection. Experimentally such an effect seems to be present with intersection 1/2.

6. *Narrowing Gaps and Hadronic Cross Sections*

It is well known that in a superconductor the energy gap decreases continuously with the number of excitations, vanishing at the critical temperature. At the critical temperature one goes through a phase transition, i.e., the state of the system becomes qualitatively different, switching to a different branch (normal state) where Cooper pairs are no longer present.

By analogy we are led to speculate that in the hadronic world when the density of excitations increases, the gap parameter might decrease. As a result, as the energy rises, for the same energy increment there would be an ever increasing number of possible excitation channels. The cross sections would then show an anomalous growth or would decrease less than expected from general arguments.

The fact that the gap parameter decreases with increasing energy and more channels contribute does not mean, of course, that at high energies we are going to observe, say, pions with a mass less than normal, because the excitations will recombine to give rise to particles with the usual masses in the asymptotic states. In any case because more channels are contributing the cross section will grow.

The fact that the experimental hadronic cross sections are growing and the e^+e^- cross section is not decreasing as expected suggests that we may very well be approaching a phase transition in hadronic production. For energies above the phase transition the hadronic excitations being produced would be qualitatively different and it is quite possible that free quarks would at last be observed.

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